On Coded Cooperative Systems: Codes, Choice of Partners and Routes

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Abstract— **In this paper, we consider the applicability of cooperative information transmission to wireless local area networks (WLAN's). As the next generation of WLAN's will utilize Orthogonal Frequency Division Multiplexing (OFDM), we consider the analysis and design of cooperative codes in the context of OFDM systems. We then consider the information transfer through wireless local area networks from an energy consumption perspective. For networks operating over a slowly Rayleigh fading channel, we consider the optimum choice of partners for cooperation and develop minimum energy cooperative routing protocols.**

I. Introduction

Information transfer through wireless local area networks (WLAN's) involves simultaneous communication among multiple source–destination pairs. Wireless local area networks may operate in infrastructure mode or as ad-hoc networks. In the infrastructure mode the coordination of these multiple communications is done via the access point. The access point processes all the signals transmitted from the sources (uplink) and forwards them to their respective destinations (downlink). In the ad–hoc mode on the other hand there is no fixed infrastructure and the terminals utilize other terminals as relays to transfer information from the source to its destination. Motivated by the diversity effects and power efficiency of communicating via relaying, recent research efforts have focused on cooperation among the terminals in the network.

Sendonaris *et al.* [1] showed that cooperating mobiles in a wireless network provide not only higher data rates, but also decreased sensitivity to channel variations. One can obtain spatial diversity through the use of this "virtual" antenna array even if the mobiles are connected via noisy links. Laneman *et al.* [2] incorporated the fact, that in practice, the relaying mobile cannot receive and transmit at the same time, and illustrated that cooperation is still beneficial in reducing the outage probability. The approach to user–cooperation through orthogonal channels via time division multiplexing is given in Figure 1. This is a suitable representation for wireless local area networks, as they operate in Time Division Duplex (TDD) mode.

Figure 2 illustrates the possible configurations of user–cooperation in a wireless local area network. User T_1 may cooperate with user T_2 to transmit its message to user T_3. Similarly, user T_2 may cooperate with user T_1 to transmit its message to user T_4. On the other hand, both user T_1 and user T_2 may cooperate to transmit their signals to a common destination, either user T_3 or T_4.

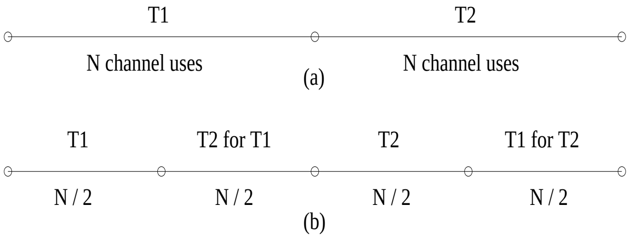

Fig. 1. Time–division channel allocations: (a) orthogonal direct transmission and (b) orthogonal cooperative diversity transmission.

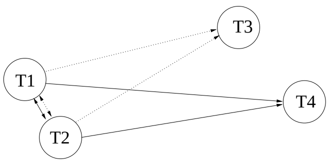

Fig. 2. Example of user–cooperation diversity in wireless local area network.

Laneman *et al.* [2] considered different protocols to achieve diversity gains such as amplify and forward or decode and forward. However, from a coding perspective these protocols resemble repetition coding, and there are more effective ways of designing channel codes. In [3], [4] we demonstrated that an overall block fading channel model [5] is appropriate in the case of user-cooperation, since the cooperating terminals observe independently faded channels towards the destination. This resulted in a framework for the design of cooperative channel codes. Cooperative coding approaches for Rayleigh fading channels have also been considered in [6], [7], [8], [9], [10].

Motivated by these results, it is of interest to consider the design of cooperative channel codes for wireless local area networks. As the next generation of WLAN's will utilize Orthogonal Frequency Division Multiplexing (OFDM), we address the problem of analysis and design of cooperative codes in the context of OFDM systems [11], [12]. We then consider the information transfer through wireless local area networks from an energy consumption perspective and develop minimum energy cooperative routing protocols. The paper is organized as follows. Section II presents cooperative coding in the context of OFDM systems. Section III provides an energy consumption perspective on coded cooperative systems and illustrates the energy savings of cooperative routing in comparison with traditional multihop routing. Section IV concludes the paper.

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II. Cooperative Coding for OFDM Systems

We consider an OFDM communication system and focus on the cooperation between two users, T_1 and T_2. The user T_i–destination channels, have L_i nonzero taps, $i = 1, 2$, respectively. The OFDM system has K subcarriers. Each channel codeword spans P adjacent OFDM words. Each user experiences slow quasi-static Rayleigh fading channel. The path gains of different users are assumed to be independent. We assume perfect channel state information at all the respective receivers.

The received signal can be expressed in the frequency domain as [13]

$$\mathbf{y}[p,k] = \boldsymbol{H}[p,k]\mathbf{x}[p,k] + \mathbf{z}[p,k] \tag{1}$$

$k = 0, \ldots, K-1$, $p = 1, \ldots, P$. $\boldsymbol{H}[p,k]$ is the matrix of complex channel frequency response at the kth subcarrier and at the pth time slot, $\mathbf{x}[p,k]$ and $\mathbf{y}[p,k]$ are the transmitted signals and the received signals, respectively, at the kth subcarrier and at the pth time slot, while $\mathbf{z}[p,k]$ is the circularly symmetric complex Gaussian noise. $\boldsymbol{H}[p,k]$ can be expressed as

$$h_{i,j}(\tau) = \sum_{l=1}^{L} \beta_{i,j}(l)\delta(\tau - \frac{n_l}{K\triangle_f}) \tag{2}$$

where $\delta(\cdot)$ is the Dirac delta function, L denotes the number of nonzero taps and $\beta_{i,j}(l)$ is the complex amplitude of the lth nonzero tap, whose delay is $\frac{n_l}{K\triangle_f}$, where n_l is an integer and $\triangle_f$ is the tone spacing of the OFDM system.

The channel frequency response between the jth transmit antenna and the ith receive antenna at the pth time slot and at the kth subcarrier, which is the (i,j)th element of the $\boldsymbol{H}[p,k]$ in (1), can be expressed as

$$H_{i,j}[p,k] = \sum_{l=1}^{L} \beta_{i,j}(l;pT)e^{-j2\pi k n_l/K} = h_{i,j}^{H}(p)\ \omega_f(k) \tag{3}$$

where $h_{i,j}(p) = [\beta_{i,j}(1), \ldots, \beta_{i,j}(L)]^H$ is the L-sized vector containing the time responses of all the nonzero taps and $\omega_f(k) = [e^{-j2\pi k n_1/K}, \ldots, e^{-j2\pi k n_L/K}]^T$ contains the corresponding DFT coefficients.

A. *Performance Analysis*

Without loss of generality, we study the cooperative coding performance gains from the perspective of node T_1. Similar results would also be obtained for node T_2. The frame error probability (FEP) can be obtained as

$$P_f^{coop} = (1 - P_f^{in})P_f^{BF} + P_f^{in}P_f^{QS} \leq P_f^{BF} + P_f^{in}P_f^{QS} \tag{4}$$

where P_f^{in} denotes the FEP of the first half codeword over the inter-user channel, P_f^{BF} denotes the FEP over the block fading channel when the cooperation takes place, and P_f^{QS} denotes the frame error probability over the quasi–static fading T_1–destination channel which the destination observes if T_2 cannot decode T_1. Let γ_1 denote the received signal–to–noise ratio at the destination corresponding to the transmission from user T_1. Similarly, let γ_2 denote the received signal–to–noise ratio at the destination corresponding to the transmission from user T_2 and γ_{in} denote the received signal–to–noise ratio at user T_2 corresponding to the transmission from user T_1.

In the block fading model resulting from cooperation, each block may have a different received signal–to–noise ratio, different number of antennas and different number of nonzero channel taps. Deriving the pairwise error probability for the block Rayleigh fading OFDM channel and utilizing the union upper bound on the frame error probability, when node T_1 transmits in cooperation with node T_2, the upper bound on the frame error probability, P_f^{coop}, is

$$\begin{aligned} P_f^{coop} \leq\ & \sum_c \sum_{e \neq c} \frac{1}{\left(\prod_{b=1}^{2} \mu_b\right) \gamma_1^{r_1} \gamma_2^{r_2}} \\ & + \left(\sum_c \sum_{e \neq c} \frac{1}{\left(\prod_{j=1}^{r_1} \kappa_j\right) \gamma_{in}^{r_{in}}} \right) \\ & \times \left(\sum_c \sum_{e \neq c} \frac{1}{\left(\prod_{j=1}^{r} \lambda_j\right) \gamma_1^{r}} \right) \end{aligned} \tag{5}$$

where r_b denotes the rank of the codeword difference matrices in the OFDM fading block b, $b = 1, 2$, and r denotes the rank of the codeword difference matrix between the two entire codewords of T_1. The μ_b's, $b = 1, 2$ are given by $\mu_1 = \prod_{i=1}^{r_1} \kappa_i$ and $\mu_2 = \prod_{i=1}^{r_2} \delta_i$, where the κ_i's and the δ_i's denote the nonzero eigenvalues of the product of the codeword difference matrix and its respective conjugate transpose for the OFDM fading block $b = 1$ and $b = 2$, respectively. The λ_i's denote the nonzero eigenvalues of the product of the codeword difference matrix between the two entire codewords and its conjugate transpose.

We consider the case when, $\gamma_1 \approx \gamma_2 \approx \gamma_{in} = \gamma$, that is all channels, including the inter–user channel, have similar quality. This assumption simplifies the diversity analysis and is quite reasonable at high signal–to–noise ratios in all channels. In this case, P_f^{coop}, can be approximately upper bounded by

$$\begin{aligned} P_f^{coop} \leq\ & \gamma^{-(L_1+L_2)} \left(\sum_c \sum_{e \neq c} \frac{1}{\left(\prod_{b=1}^{2} \mu_b\right)} \right) \\ & + \gamma^{-(L_1+L_{in})} \left(\sum_c \sum_{e \neq c} \frac{1}{\left(\prod_{j=1}^{L_{in}} \kappa_j\right)} \right) \\ & \times \left(\sum_c \sum_{e \neq c} \frac{1}{\left(\prod_{j=1}^{L_1} \lambda_j\right)} \right) \end{aligned} \tag{6}$$

Here we have assumed that the codeword difference matrices in both fading blocks, $b = 1, 2$ are of full rank. Let $k = \min\{L_{in}, L_2\}$. At high signal–to–noise ratios, we have the following approximation

$$P_f^{coop} \approx \mathcal{K}_1\ \gamma^{-(L_1+k)} \tag{7}$$

where the term $\mathcal{K}_1$ represents the coding parameters. This means that when all links have the same average quality, the diversity order achieved through cooperative coding depends on $k = \min\{L_2, L_{in}\}$, as indicated by the exponent of the signal–to–noise ratio.

Next, we focus on the case when the inter–user channel is very good, i.e., it has a very high signal–to–noise ratio. This could represent the scenario when the two partners are located very close to each other. This means that P_f^{in} is small and we simply have $P_f^{coop} \approx P_f^{BF}$. Hence,

$$\begin{aligned} P_f^{coop} &\approx \gamma^{-(r_1+r_2)} \sum_c \sum_{e\neq c} \left(\prod_{b=1}^{2} \mu_b\right)^{-1} \qquad (8) \\ &= \mathcal{K}_2\, \gamma^{-(r_1+r_2)} \qquad (9) \end{aligned}$$

where $\mathcal{K}_2 = \sum_c \sum_{e\neq c} \left(\prod_{b=1}^{2} \mu_b\right)^{-1}$. We observe that when the inter-user channel quality is very good, the full user cooperation diversity of (r_1+r_2) is achieved. In the scenario when we have maximum transmit diversity $r_1 = L_1$ and $r_2 = L_2$, the maximum user–cooperation diversity that can be achieved is $(r_1 + r_2) = (L_1 + L_2)$.

Finally, when the inter-user channel quality is very poor, the inter-user channel signal-to-noise ratio, γ_{in}, will be lower than the signal-to-noise ratio of the user-destination channel. We can assume that $\gamma_{in}^{-r_{in}} \leq C_{in}$, for all signal-to-noise ratios of interest. Hence, P_f^{coop} is upper bounded by the term $P_f^{in} P_f^{QS}$, yielding

$$P_f^{coop} \leq \frac{\mathcal{K}_3}{C_{in}} \gamma_1^{-r_1} \tag{10}$$

where $\mathcal{K}_3 = \min_{(c,e)}\{(\prod_{j=1}^{r_1} \lambda_j \prod_{j=1}^{r_{in}} \mu_j)^{-1}\}$. In this case, the diversity level is only r_1. This is the same diversity level achieved by T_1 when there is no cooperation. However, there is still some coding gain, as indicated by the eigenvalue product, compared to the conventional OFDM system.

B. Numerical Examples

We next present the simulation results for the coded cooperative OFDM system. We assume that the OFDM system has $K = 128$ subcarriers. We consider the constraint length 7 convolutional code (133,171,117,165) and BPSK modulation. We assume maximum likelihood detection at the destination. The frame size is 256 bits and each codeword spans $P = 2$ OFDM words. Both user-destination channels are assumed to have two taps, namely $L_1 = L_2 = 2$.

Fig. 3 illustrates the frame error rate (FER) performance comparison between the non-cooperative case and the cooperative case for different inter-user channel qualities. Both user-destination channels have similar quality. We observe that when the inter-user channel quality is very good, we achieve full diversity. The gain over the single user performance is about 6.5 dB at a FER of 10^{-3}. We note that even in the case when the inter-user channel FER is 0.5, we still obtain about 2 dB improvement at a FER of 10^{-3} as compared to the non-cooperative case.

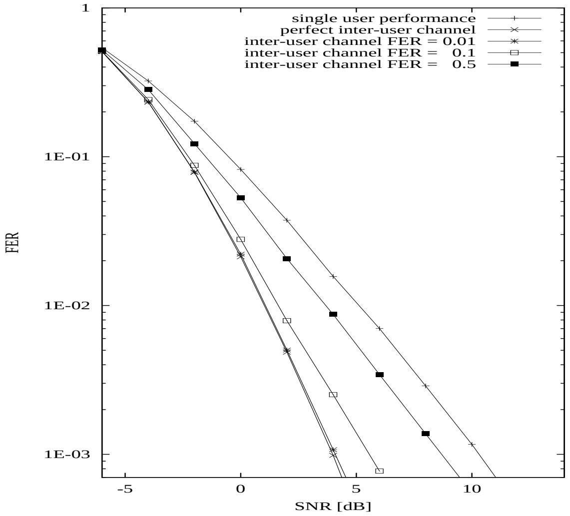

Fig. 3. Single user performance vs. two user cooperation, for different inter-user channel qualities.

Next we consider the scenario when one of the users has much better channel to the destination than the other partner. Fig. 4 illustrates the FER performance for both users in this asymmetric scenario. We assume that user T_1 has better channel quality to the destination, i.e., its SNR is fixed at 10.3 dB, resulting in a FER of 10^{-3}. We observe the performance of both users as we vary the SNR of user T_2. The inter-user channel frame error rate is 10^{-1}. From Fig. 4 it can be observed that both users benefit from cooperation. User T_1 achieves the FER of 10^{-3} when the SNR of user T_2 is about -4 dB. At higher SNRs, its performance is even better than in the non-cooperative case. User T_2 also has significant gains, as it improves its performance by about 5 dB with respect to the non-cooperative case.

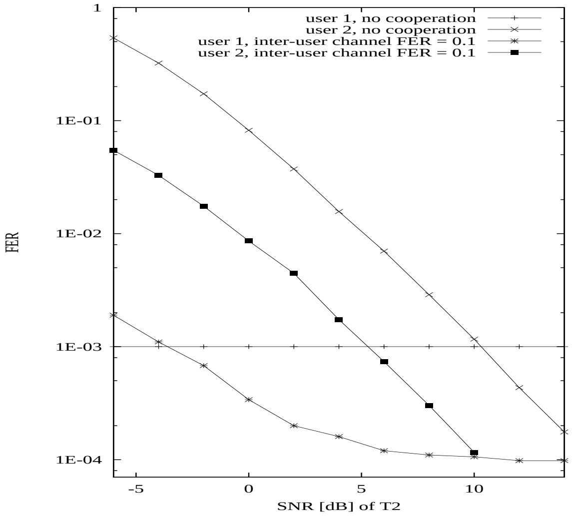

Fig. 4. Single user performance versus two user cooperation, for two users with different channel qualities.

III. AN ENERGY CONSUMPTION PERSPECTIVE ON CODED COOPERATIVE SYSTEMS: CHOICE OF PARTNERS AND ROUTES

In the development of the energy consumption model, we focus on the quasi-static Rayleigh fading channel model. We begin with the frame error probability expression for cooperative systems given in Eq. 4. However, as the use of bounds on the frame error probability leads to computationally intensive solutions, we consider the use of tight bounds on the pairwise error probabilities with suitably defined offset factors that relate the pairwise error probability (PEP) to the frame error probability. This is similar to the approach considered in [14]. We utilize the fact that for carefully designed channel codes, i.e., codes that achieve certain degree of diversity, the PEP and FER curves are parallel for moderate to high signal–to–noise ratios. The tight upper bound on the probability of confusing two codewords $\underline{c}$ with $\underline{e}$ for the cooperative block fading channel is [16]

$$PEP(\underline{c} \to \underline{e}) \leq \frac{\kappa}{\prod_{i=1}^{2} d_i^2 \frac{\gamma_i}{4}} \tag{11}$$

where d_i is the Euclidean distance between the portions of the two codewords residing in block i and γ_i is the average received SNR for block i, $i = 1, 2$, and $\kappa := \frac{1}{2}\prod_{i=1}^{2}\left(1 - \frac{1}{2i}\right) = \frac{3}{16}$ for this two fading blocks case.

Utilizing the offsets Q_{qs} and Q_{in} to relate the FER and the PEP, the FER's for quasi-static non-cooperative channel and inter-user channel can be obtained as

$$P_f^{QS} \leq \frac{Q_{qs}}{(d_1^2 + d_2^2)\gamma_1}, \tag{12}$$

$$P_f^{in} \leq \frac{Q_{in}}{d_1^2 \gamma_{in}}\cdot \tag{13}$$

Similarly, for the block fading case, utilizing the offset factor Q_{bf}^2, the FER can be expressed as

$$P_f^{BF} \leq \frac{3Q_{bf}^2}{d_1^2 d_2^2 \gamma_1 \gamma_2}\cdot \tag{14}$$

Hence, from Eq. 4, it follows that the FER for a coded cooperative system can be upper bounded as

$$P_f^{coop} \leq \frac{3Q_{bf}^2}{d_1^2 d_2^2 \gamma_1 \gamma_2} + \frac{Q_{qs} Q_{in}}{d_1^2 (d_1^2 + d_2^2) \gamma_{in} \gamma_1}\cdot \tag{15}$$

According to the link budget relationship [15], the transmission power, P_t, can be written as

$$P_t = \frac{\gamma N_0 R_b D^{\alpha}}{K_g R_c} \tag{16}$$

where γ is the required average received SNR for a given FER requirement, R_b is the bit rate, R_c is the coding rate, D is the normalized transmission distance, α is the path loss exponent which is determined by the environment, N_0 is the thermal noise power spectral density and K_g is the channel power gain. The thermal noise power spectral density can be written as [15], $N_0 = FkT_0$, where F is the system noise figure, $k = 1.38 \times 10^{-23} J/K$ is the Boltzmann's constant and T_0 is the room temperature in kelvins. The channel power gain is [15], $K_g = (c/4\pi f_c)^2$, where c is the speed of light and f_c is the carrier frequency.

Hence, approximating the tight upper bound with an equality, the energy consumption expression for coded cooperative systems is $E_b^{coop} = \frac{P_t}{R_b}$. Hence, in the case when $\alpha = 2$, we obtain

$$E_b^{coop} = \frac{N_0 D_1 \sqrt{3Q_{bf}^2 (d_1^2 + d_2^2)^2 D_2^2 + Q_{qs} Q_{in} (d_1^2 + d_2^2) {d_2}^2 D_{in}^2}}{K_g R_c \sqrt{P_f^{coop}} (d_1^2 + d_2^2) d_1 d_2}. \tag{17}$$

For non-cooperative quasi-static channel, the energy consumption per bit is given by

$$E_b^{no-coop} = \frac{N_0 D_1^2 Q_{qs}}{K_g R_c P_f^{no-coop} (d_1^2 + d_2^2)}\cdot \tag{18}$$

This energy-based approach is equivalent to the one which uses the frame error probability as a metric. In other words, the minimum energy route given a target frame error probability is the same as the lowest frame error probability route given the energy available at transmit side. Rewriting Eq. 17 as

$$(E_b^{coop})^2 = \frac{D_1^2 N_0^2 \left[3Q_{bf}^2 (d_1^2 + d_2^2)^2 D_2^2 + Q_{qs}^2 d_1^2 d_2^2 D_{in}^2\right]}{P_f^{coop} \left[d_2 (d_1^3 + d_1 d_2^2) K_g R_c\right]^2}. \tag{19}$$

we observe that for a given target P_f^{coop} value, we can calculate and compare the different E_b^{coop} values and choose the candidate with the lowest E_b^{coop} value to cooperate. In fact, the difference in the obtained E_b^{coop} values when choosing different partners to cooperate lies in the following portion of the expression in Eq. 19 and is given in terms of D_2 and D_{in},

$$\zeta = 3Q_{bf}^2 (d_1^2 + d_2^2)^2 D_2^2 + Q_{qs}^2 d_1^2 d_2^2 D_{in}^2 \cdot \tag{20}$$

Hence, while Eq. 17 measures the transmission energy consumption and provides insight into the energy and reliability trade off, the partner choice and cooperative route determination can be simplified by using Eq. 20 as a metric, that is, choosing a partner that minimizes ζ, while ensuring that the resulting E_b^{coop} value is less than $E_b^{no-coop}$. We illustrate these observations through numerical examples in the next section.

A. Numerical Examples

We consider a wireless local area network operating in an ad–hoc mode. The multi–hop route from the source to the destination is already established. There are ten nodes along this route. Each node is aware of the locations of its neighboring nodes. The network is of low mobility and the distances between the communicating nodes remain unchanged on a time scale larger than the transmission time. For simplicity, we focus on the rate 1/4 [5,7,5,7] convolutional code and BPSK modulation. We assume the system noise figure $F = 6$ dB and the temperature $T_0 \approx 300$ K. Hence, the noise spectral density $N_0 = 1.6 \times 10^{-20}$ W/Hz. For a 2.4 GHz operating frequency, the channel power gain $K_g = (c/4\pi f_c)^2 \approx 10^{-4}$. Without loss of generality, let us consider the first hop along the route. The energies for all five possible cooperating pairs are calculated according to Eq. 20. We choose a partner among all candidates that minimizes E_b^{coop}. Among the five pairs, cooperating with the node that has the distance parameters $\{D_1, D_2, D_{in}\}=\{2.3254, 1.9721, 1.7768\}$ will consume the least amount of energy. Thus, this relay is chosen as a partner to cooperate. Following this procedure, we obtain a list of partners for the nine hops along the route. The results of partner choice are shown in Fig. 5.

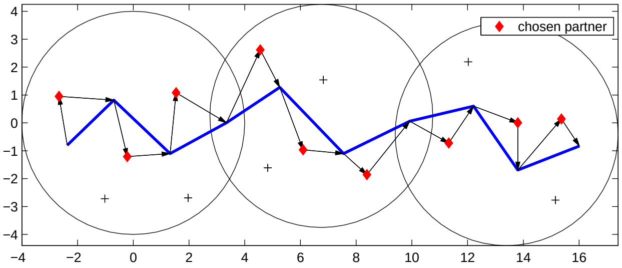

Fig. 5. Cooperative routing with optimum choice of partners.

Finally, in Fig. 6, we present the average energy savings per hop, that coded cooperation provides with respect to the non-cooperative coded multihop transmission in slow Rayleigh fading channels. As expected the savings are more pronounced for lower values of the target frame error rate (FER), which is a suitable scenario for data communications.

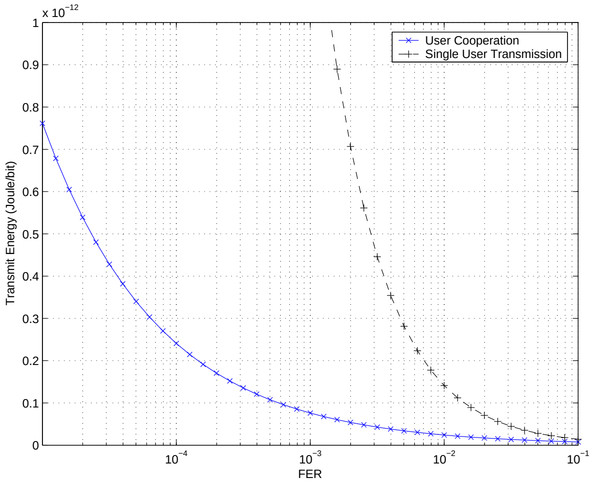

Fig. 6. Average energy savings per hop.

IV. Conclusions

We considered the applicability of cooperative information transmission to wireless local area networks (WLAN's). We provided an analysis and design of cooperative codes in the context of OFDM systems and demonstrated their viability for WLAN's utilizing Orthogonal Frequency Division Multiplexing. We then considered the information transfer through wireless local area networks from an energy consumption perspective. We obtained simple rules for choosing partners among the neighboring nodes in the network and utilized them to develop minimum energy cooperative routing algorithms. We illustrated significant energy savings for slow Rayleigh fading channels.

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