A Cross-Layer Approach to Decentralized Detection in Sensor Networks with Noisy Communication Links and Multiple Observations†

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***Abstract*— This paper presents a general approach to distributed detection in sensor networks in scenarios with noisy communication links between the sensors and the fusion center (or access point, AP). The sensors are independent and observe a common phenomenon. While in most of the literature the performance metrics usually considered are missed detection and false alarm probabilities, in this paper we follow a Bayesian approach for the evaluation of the probability of *decision error* at the AP. We first derive an optimized fusion rule at the AP in a scenario with ideal communication links. We then consider the presence of noisy links and model them as binary symmetric channels (BSCs). In this case, we show that if the noise intensity is above a critical level (i.e., the cross-over probability of the BSC is above a critical value), the probability of decision error at the AP reduces when the AP selectively discards the information transmitted by the sensors with noisy links. We will also show that use of multiple observations at the sensors can be traded for increased robustness against channel impairments in the communication links.**

I. Introduction

Distributed detection has been an active research field for a long time [1]. In particular, several approaches have been proposed to study this problem, in the realms of information theory [2], target recognition [3], [4], and several other areas. The increasing interest, over the last decade, for sensor networks, has spurred a significant research activity on distributed detection techniques in this context [5]–[8].

In recent years, wireless sensor networks are becoming more common, as, for example, in terrain monitoring applications [9]. In a wireless communication scenario, links between sensors and the access point (AP) are likely to be faded [10]. In this case, most of the results proposed in the literature are not immediately applicable, since they are based on the assumption that communication links between sensors and AP are "ideal," i.e., the information transmitted by sensors is received correctly by the AP. The characteristics (in terms of capacity) of the multiple access radio channel in wireless sensor networks are taken into account in [11], where optimal configurations for decentralized detection are studied.

In this paper, we first revisit the basic principles of distributed detection with binary decisions at the sensors. In order to model a scenario where some of the links between sensors and AP are non-ideal (e.g., some of links in a wireless sensor network may be faded), we assume that a link can be modeled as a binary symmetric channel (BSC) [12]. We show that selective elimination of noisy links may lead to a performance improvement when the *cross-over* (or *bit-flipping*) probability of the BSC increases. In particular, for each value of the common signal-to-noise ratio (SNR) at the sensors we determine a critical bit-flipping probability which discriminates between two network operating regimes: for values of the bit-flipping probability above this value, the best performance is obtained when the AP excludes the sensors with noisy links. We will also show that the use of multiple observations at each sensor, as proposed in [13], increases the robustness of the network against impairments in the communication links. More precisely, increasing the number of observations at the sensor increases the value of the critical bit-flipping probability discriminating for selective exclusion of the sensors with noisy links. A strategy based on selective exclusion of sensors with noisy links could be obtained by using a clever medium access control (MAC) protocol at the AP. Therefore, our results suggest that the use of a *cross-layer approach* to the design of sensor networks with unreliable communication links (e.g., wireless sensor networks) may be an advantage.

This paper is structured as follows. In Section II, we provide the reader with preliminaries on distributed detection principles, referring to a classical distributed detection scheme, with *parallel* schedule. In Section III, the presence of noisy links, modeled as BSCs, is considered, and the corresponding sensor network performance, with single and multiple observations at the sensors, is analyzed. Conclusions and future research directions are presented in Section IV.

II. Preliminaries on Distributed Detection

We consider a classical sensor network scenario where all sensors are connected to a single AP [5]. Two main approaches for combining the information gathered by multiple sensors have been proposed.

- The first approach is referred to as *centralized*: all sensors observations are transmitted to a central processor that performs a global decision.

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- The second approach is referred to as *decentralized*: each sensor makes a local decision and transmits it to a fusion processor, i.e., the AP, which makes the final decision, by applying a suitable *fusion rule*.

In this paper, all sensors make *at least one* observation of a common binary phenomenon. In other words, we consider the binary hypothesis signal detection problem [14], with statistically independent observations from sensor to sensor. We will refer to the two hypotheses as H_1 and H_0, respectively. The true hypothesis will be simply denoted as H. We will assume that the two hypotheses are equally likely. The extension of this work to the case of correlated sensors [15] is currently under investigation.

We assume that there are N sensors. The discrete-time observation at the i-th sensor can be expressed as

$$r_i = y_i + n_i \tag{1}$$

where

$$y_i \triangleq \begin{cases} 0 & \text{if} \quad H_0 \\ s & \text{if} \quad H_1 \end{cases}$$

with $i = 1, 2, \ldots, N$. Assuming that the noise samples $\{n_i\}$ are independent and identically distributed with the same Gaussian distribution $\mathcal{N}(0, \sigma^2)$, the common signal-to-noise ratio (SNR) at each sensor can be defined as follows:

$$\text{SNR}_{\text{sensor}} \triangleq \frac{[\text{E}\{y_i|H_1\} - \text{E}\{y_i|H_0\}]^2}{\sigma^2} = \frac{s^2}{\sigma^2}. \tag{2}$$

For the sake of notation simplicity, in the remainder of the paper we will assume that $\sigma^2 = 1$, so that $\text{SNR}_{\text{sensor}} = s^2$. We also assume that the SNR is the same at all sensors, i.e., the sensors are equivalent.

In a classical distributed detection parallel scheme, each sensor makes an observation of the common phenomenon, decides for one of the two hypotheses, and then sends its binary decision, denoted as u_i, to the AP. In general, the decision rule at each sensor (common for all sensors) can be written as $u_i = \gamma(r_i)$, where $\gamma(\cdot)$ is a suitable decision function. Usually, the communication link between each sensor and the AP is *ideal*, i.e., the AP receives correctly the bit transmitted by each sensor. In order to make a decision, the i-th sensor compares the observation r_i with a threshold value τ and computes a binary decision, denoted by u_i, as follows:

$$u_i = \gamma(r_i) = \begin{cases} 1 & \text{if} \quad r_i < \tau \\ 0 & \text{if} \quad r_i > \tau. \end{cases}$$

Obviously, $\gamma(r_i) = U(r_i - \tau)$, where $U(\cdot)$ is the unitary step function. It is possible to show that this decision rule is equivalent to a local likelihood ratio test [16]. In [17], it is shown that selecting the same value of τ for all sensors is an asymptotically (for large values of N) optimal choice for minimizing the probability of incorrect decision. Moreover, in [17] the author shows that even selecting the same value of τ for a relatively small number N of sensors leads a negligible performance loss with respect to optimal threshold selection among the sensors. Motivated by this observation, in the remainder of this paper we will assume that the threshold value τ for local decision is the same for all sensors.

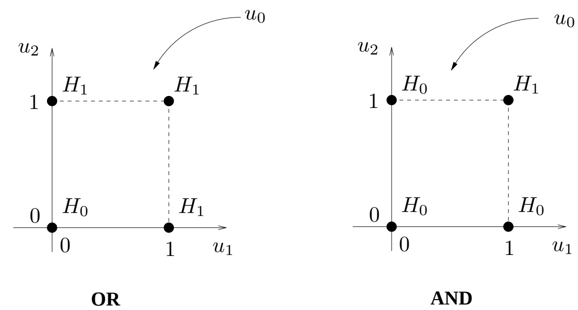

Fig. 1. Decision regions for majority-like fusion rules in the case with $N = 2$ sensors: OR (left) and AND (right). In the axes there are the local decisions at the two sensors (u_1 and u_2), while within the diagram there is the final decision at the AP (u_0).

Once all sensors have made their local decisions $\{u_i\}$, the AP receives an array of N binary values, and makes a final decision u_0 according to a fusion rule $u_0 = \Gamma(u_1, \ldots, u_N)$. As shown in the literature, the fusion rule must be based on a binary monotonic increasing function of the decisions array of length N [5]. Given N, even if there are 2^{2^N} possible fusion rules, one can limit herself/himself at investigating only binary monotonic increasing functions [3], [5]. Under the assumption that the SNR is the same at all sensors, these fusion rules can be given the following general *majority-like* expression [18]:

$$\Gamma(u_1, \ldots, u_N) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{N} u_i \geq k \\ 0 & \text{if} \quad \sum_{i=1}^{N} u_i < k \end{cases} \tag{3}$$

where $k = 1, \ldots, N$. In general, if $k = 1$ the OR fusion rule is obtained, while if $k = N$ the AND fusion rule is obtained. In a network with $N = 2$ sensors, only the OR and AND fusion rules are possible and they are depicted in Fig. 1.

Provided that the fusion rule is in the form given by (3), the key problem consists in determining the value of k that minimizes the probability of decision error, defined as

$$P_\text{e} \triangleq P\{u_0 \neq H\}.$$

Based on our assumption of equally likely hypotheses ($P(H_0) = P(H_1) = 1/2$), the probability of error can be written as

$$P_\text{e} = \frac{1}{2}P(u_0 = H_0|H_1) + \frac{1}{2}P(u_0 = H_1|H_0). \tag{4}$$

We now first derive the expression for the probability of decision error at the AP in a simple scenario with $N = 2$ sensors, and we thus generalize it to the case with $N > 2$ sensors.

A. Probability of Error with $N = 2$ Sensors

By applying the total probability theorem and recalling the independence among the sensors, after proper manipulations the probability of error (4) can be expressed as

$$P_{\rm e} = \begin{cases} \frac{1}{2}[1-\Phi(\tau)]^2 + \Phi(\tau - s) - \frac{1}{2}\Phi^2(\tau - s) \\ \qquad \text{AND rule} \\ \frac{1}{2}[1-\Phi(\tau)]^2 + [1-\Phi(\tau)]\Phi(\tau) + \frac{1}{2}\Phi^2(\tau - s) \\ \qquad \text{OR rule} \end{cases} \quad (5)$$

where $\Phi(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}\,\mathrm{d}y$. The expressions in (5) depend on the threshold τ and on $s = \sqrt{\text{SNR}}$. For a given value of s, one can find numerically the best threshold value by minimizing the probability of error with respect to τ. The best numerical value for τ depends on the decision rule (either AND or OR). However, it can be shown that the corresponding probability of error is the same in both cases: this is due to the symmetry of the rules, with respect to all possible errors, conditionally on the hypothesis.

B. Probability of Error with $N \geq 2$ Sensors

In the general case with $N \geq 2$ sensors, the two terms at the right side of (4) can be written as

$$\begin{aligned} P(u_0 = H_0|H_1) &\\ &= P\{\text{less than } k \text{ sensors decide for } H_0|H_1\} \\ &= \sum_{i=0}^{k-1}\binom{N}{i}[1-\Phi(\tau - s)]^i\Phi^{N-i}(\tau - s) \qquad (6) \\ P(u_0 = H_1|H_0) &\\ &= P\{\text{at least } k \text{ sensors decide for} H_1|H_0\} \\ &= \sum_{i=k}^{N}\binom{N}{i}[1-\Phi(\tau)]^i\Phi^{(N-i)}(\tau). \qquad (7) \end{aligned}$$

Therefore,

$$\begin{aligned} P_{\rm e} &= \frac{1}{2}\sum_{i=0}^{k-1}\binom{N}{i}[1-\Phi(\tau - s)]^i\Phi^{N-i}(\tau - s) \\ &+ \frac{1}{2}\sum_{i=k}^{N}\binom{N}{i}[1-\Phi(\tau)]^i\Phi^{N-i}(\tau). \qquad (8) \end{aligned}$$

The behavior of the probability of error, as a function of the threshold value τ, is shown in Fig. 2, in the case with $\text{SNR}_{\text{sensor}} = 0\,\text{dB}$. As one can observe from Fig. 2, for each decision rule the probability of error is a quasi-convex function of τ and has an absolute minimum. Numerically, one can characterize the absolute minimum depending on the value of N.

- N odd: the optimal value of τ is $s/2$; the best fusion rule is the *majority rule*, i.e., $k = \lfloor N/2 \rfloor + 1$.
- N even: between the optimal value for the threshold τ and $s/2$ there is an offset that, in general, depends on (i) the number of sensors N , (ii) the sensor SNR s^2 and (iii) the fusion rule. In particular, the best fusion rules are obtained selecting $k = N/2+1$ (i.e., adopting a majority rule) or $k = N/2$. For both fusion rules, by properly selecting the threshold value τ the probability of decision error is the same.

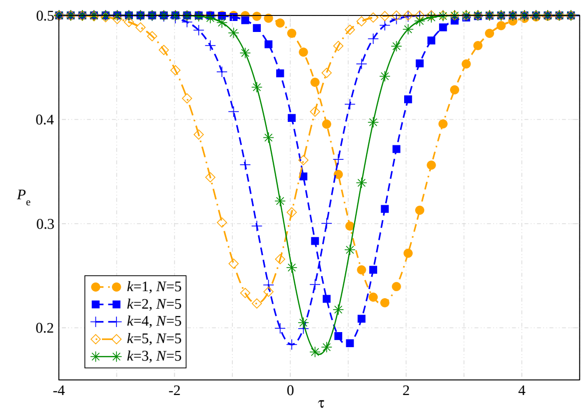

Fig. 2. Probability of error, as a function of the threshold value τ, in a scenario with $N = 5$ sensors and $\text{SNR}_{\text{sensor}} = 0$ dB. Various values of k, corresponding to different fusion rules, are shown.

As intuitively expected, it can be shown (the results are not reported here for lack of space) that increasing the number of sensors N and choosing the corresponding optimal fusion rule, the performance (in terms of $P_{\rm e}$) improves drastically.

III. Sensor Networks with Noisy Communication Links

While all previous results apply to a sensor network scenario where the communication links between sensors and AP are ideal, in a realistic scenario (e.g., a *wireless* sensor network) it might happen that these links are noisy (e.g., they are affected by *fading*). Studying such a scenario is difficult, since the presence of fading might create correlations among the sensors [10]. The analysis and optimization of wireless sensor networks is, therefore, a complicated problem. In order to derive significant insights into this problem, in the following we consider a simple model for a noisy communication link.

A noisy link between a sensor and the AP is modeled as a BSC with parameter p, corresponding to the cross-over probability[1] [19]. In other words, the bit transmitted by the sensor has a probability p of being "flipped." The parameter p will depend on the specific characteristics of the sensors-AP communication links (e.g., modulation format, presence of channel coding, presence of fading, detection strategy at the AP, etc.). Assuming binary hard decision at each sensor, if u_i is the decision sent by the i-th sensor, the AP will receive the following information:

$$u_i^{\text{received}} = \begin{cases} u_i & \text{with probability } 1-p \\ 1-u_i & \text{with probability } p. \end{cases}$$

We now extend the derivation of the probability of error proposed in Section II in order to encompass the presence

[1] We remark that the sensor SNR, i.e., $\text{SNR}_{\text{sensor}} = s^2$, is the SNR at each sensor relative to the local detection of the common phenomenon (or state of nature). Each communication link between a sensor and the AP can be characterized by an SNR at the AP. In this paper, however, we do not explictly consider the communication link SNR, since we concisely describe the communication link as a BSC, which is completely characterized by the single parameter p.

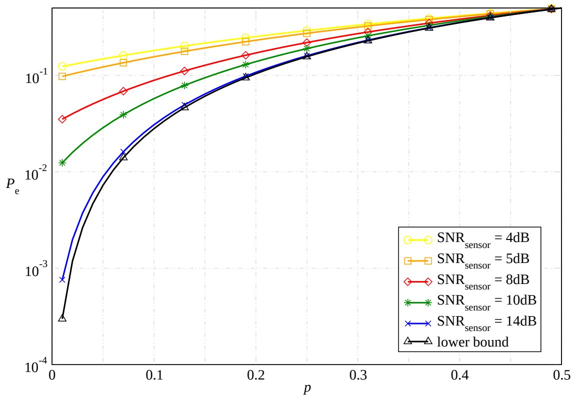

Fig. 3. Probability of error, as a function of p, for different values of the sensor SNR. The number of sensors is $N = 3$. The curve labeled "lower bound" corresponds to the theoretical limit with $\mathrm{SNR}_{\mathrm{sensor}} = \infty$.

of bit flipping. More precisely, we want to evaluate the final probability of error (4) in a sensor network with noisy communication links. We consider a majority-like fusion rule as described in Section II-A, with optimized values of k and τ. We first consider a scenario where all N links are noisy. Then, we generalize the obtained results to the case where $d \leq N$ links are noisy.

A. Sensor Networks with All Noisy Communication Links

After proper algebraic manipulations, it is possible to show that the first term in the expression (4) for the probability of decision error can be written as

$$\begin{aligned} P(u_0 = H_0|H_1) &= P\{i < k \text{ sensors for } H_1|H_1\} \\ &= \sum_{i=0}^{k-1} \binom{N}{i} P_{\mathrm{c1}}^{i} P_{\mathrm{e1}}^{N-i} \end{aligned} \tag{9}$$

where $P_{\mathrm{c1}} = (1-p)[1-\Phi(\tau - s)] + p\Phi(\tau - s)$ and $P_{\mathrm{e1}} = 1 - P_{\mathrm{c1}}$. Similarly, the second term of (4) can be written as

$$\begin{aligned} P(u_0 = H_1|H_0) &= P\{i \geq k \text{ sensors for } H_1|H_0\} \\ &= \sum_{i=k}^{N} \binom{N}{i} P_{\mathrm{e2}}^{i} P_{\mathrm{c2}}^{N-i} \end{aligned} \tag{10}$$

where $P_{\mathrm{e2}} = (1-p)[1-\Phi(\tau)] + p\Phi(\tau)$ and $P_{\mathrm{c2}} = 1 - P_{\mathrm{e2}}$.

The probability of error (4) can then be evaluated numerically, by using the derived expressions (9) and (10). In particular, the probability of error depends on (i) the decision threshold value τ at the sensors, (ii) the sensor SNR s^2, and (iii) the probability of bit-flipping p.

In Fig. 3, the probability of error is shown as a function of the bit-flipping probability p, for various values of $\mathrm{SNR}_{\mathrm{sensor}}$, in a scenario with $N = 3$ sensors. As one can observe, regardless of the sensor SNR, for increasing values of p the probability of error becomes unacceptable. The *lower bound* corresponds to a theoretical case where the sensor SNR is infinite. This lower bound, denoted as $P_{\mathrm{e-lb}}(p)$ (to underline its dependence on the bit-flipping probability p), can be given the following analytical expression:

$$\begin{aligned} P_{\mathrm{e-lb}}(p) &= \lim_{s \to \infty} P_{\mathrm{e}} \\ &= \frac{1}{2}\left[\sum_{i=0}^{k-1}\binom{N}{i}(1-p)^i p^{N-i} + \sum_{i=k}^{N}\binom{N}{i} p^i (1-p)^{N-i}\right]. \end{aligned}$$

From the results shown in Fig. 3, one can conclude that increasing the sensor SNR beyond a critical threshold does not lead to any significant performance improvement, for any value of p. This might have practical implications on the design of sensors, in terms of their detection accuracy. In other words, one should not increase the sensor sensitivity without limit, but, rather, should find the critical sensitivity at which the ultimate theoretical performance is practically achieved.

B. Sensor Networks with a Generic Number of Noisy Links

We now extend the previous analysis to encompass the case with a generic number $d \leq N$ of noisy links and, consequently, $N-d$ ideal links. The fusion rule is the majority-like rule given in (3), with optimized value of k.

In order to evaluate the probability of error, we first compute the first term on the right-hand side of (4), i.e., $P(u_0 = H_1|H_0)$. Let us denote by $d_e \leq d$ the number of bit-flipped links associated to sensors in error, i.e., sensors which decide for H_1 when H_0 has happened, and by $i_e \leq N-d$ the number of ideal links associated to sensors in error, i.e., sensors which decide for H_1 when H_0 has happened. With these definitions, the AP *might make*[2] a final erroneous decision if $d_e + i_e \geq k$, with $d_e \in \{0, \ldots, d\}$ and $i_e \in \{0, \ldots, N-d\}$. Depending on the relations between the integers N, k and d, one can distinguish the following four cases, respectively: (a) $d \geq k$, $N-d \geq k$, (b) $d \geq k$, $N-d < k$, (c) $d < k$, $N-d < k$ and (d) $d < k$, $N-d \geq k$. After tedious manipulations, the expressions for $P(u_0 = H_1|H_0)$ in the considered four cases are shown in Table I, where

$$P_{\mathrm{eH_0}} \triangleq P(u_0 = 1|H_0, p = 0) = 1 - \Phi(\tau)$$

and $P_{\mathrm{cH_0}} = 1 - P_{\mathrm{eH_0}}$.

We then consider the second term on the right-hand side of (4), i.e., $P(u_0 = H_0|H_1)$. In this case, the AP makes a final decision error when $n \leq k-1$ sensors decide for H_1. Let us define by d_c and i_c the number of sensors not in error (i.e., they decide for H_0 even if H_1 has happened) connected with noisy and ideal links to the AP, respectively. A final decision error *might happen* if $d_c + i_c \leq k-1$, with $d_c \in \{0, \ldots, d\}$ and $i_c \in \{0, \ldots, N-d\}$. As for the computation of $P(u_0 = H_1|H_0)$, four possible cases can be distinguished, depending on the values of N, k and d: (a) $d \leq k-1$, $N-d \leq k-1$, (b) $d \leq k-1$, $N-d > k-1$, (c) $d > k-1$, $N-d > k-1$ and (d) $d > k-1$, $N-d \leq k-1$, respectively. Extending the previous analysis, one obtains the analytic expressions for $P(u_0 = H_0|H_1)$ shown in Table II, where

$$P_{\mathrm{eH_1}} \triangleq P(u_0 = H_0|H_1) = \Phi(\tau - s)$$

[2] The reader should observe that if a sensor is in error and the bit transmitted to the AP is flipped, then the bit actually received by the AP is correct.

TABLE I

ANALYTIC EXPRESSIONS OF $P(u_0 = H_1|H_0)$ IN THE FOLLOWING FOUR CASES: (A) $d \geq k$, $N-d \geq k$, (B) $d \geq k$, $N-d < k$, (C) $d < k$, $N-d < k$ AND (D) $d < k$, $N-d \geq k$.

Case	$P(u_0 = H_1\|H_0)$
(a)	$\sum_{d_e=0}^{k} \left[\binom{d}{d_e} P_{\mathrm{e}_2}^{d_e} P_{\mathrm{c}_2}^{d-d_e} \sum_{i_e=k-d_e}^{N-d} \binom{N-d}{k-d_e} P_{\mathrm{eH}_0}^{i_e} P_{\mathrm{cH}_0}^{N-d-i_e}\right] + \sum_{d_e=k+1}^{d} \binom{d}{d_e} P_{\mathrm{e}_2}^{d_e} P_{\mathrm{c}_2}^{d-d_e} \cdot U(d-k-1)$
(b)	$\sum_{d_e=k+d-N}^{k} \left[\binom{d}{d_e} P_{\mathrm{e}_2}^{d_e} P_{\mathrm{c}_2}^{d-d_e} \sum_{i_e=k-d_e}^{N-d} \binom{N-d}{k-d_e} P_{\mathrm{eH}_0}^{i_e} P_{\mathrm{cH}_0}^{N-d-i_e}\right] + \sum_{d_e=k+1}^{d} \binom{d}{d_e} P_{\mathrm{e}_2}^{d_e} P_{\mathrm{c}_2}^{d-d_e} \cdot U(d-k-1)$
(c)	$\sum_{d_e=k+d-N}^{d} \left[\binom{d}{d_e} P_{\mathrm{e}_2}^{d_e} P_{\mathrm{c}_2}^{d-d_e} \sum_{i_e=k-d_e}^{N-d} \binom{N-d}{k-d_e} P_{\mathrm{eH}_0}^{i_e} P_{\mathrm{cH}_0}^{N-d-i_e}\right]$
(d)	$\sum_{d_e=0}^{d} \left[\binom{d}{d_e} P_{\mathrm{e}_2}^{d_e} P_{\mathrm{c}_2}^{d-d_e} \sum_{i_e=k-d_e}^{N-d} \binom{N-d}{k-d_e} P_{\mathrm{eH}_0}^{i_e} P_{\mathrm{cH}_0}^{N-d-i_e}\right]$

TABLE II

ANALYTIC EXPRESSIONS OF $P(u_0 = H_0|H_1)$ IN THE FOUR CASES CORRESPONDING TO (A) $d \leq k-1$, $N-d \leq k-1$, (B) $d \leq k-1$, $N-d > k-1$, (C) $d > k-1$, $N-d > k-1$ AND (D) $d > k-1$, $N-d \leq k-1$.

Case	$P(u_0 = H_0\|H_1)$
(a)	$\sum_{d_c=k+d-N}^{d} \left[\binom{d}{d_e} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c} \sum_{i_c=0}^{k-1-d_c} \binom{N-d}{i_c} P_{\mathrm{cH}_1}^{i_c} P_{\mathrm{eH}_1}^{N-d-i_c}\right] + \sum_{d_c=0}^{k-1+d-N} \binom{d}{d_c} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c}$
(b)	$\sum_{d_c=0}^{d} \left[\binom{d}{d_e} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c} \sum_{i_c=0}^{k-1-d_c} \binom{N-d}{i_c} P_{\mathrm{cH}_1}^{i_c} P_{\mathrm{eH}_1}^{N-d-i_c}\right]$
(c)	$\sum_{d_c=0}^{k-1} \left[\binom{d}{d_e} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c} \sum_{i_c=0}^{k-1-d_c} \binom{N-d}{i_c} P_{\mathrm{cH}_1}^{i_c} P_{\mathrm{eH}_1}^{N-d-i_c}\right]$
(d)	$\sum_{d_c=k+d-N}^{k-1} \left[\binom{d}{d_e} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c} \sum_{i_c=0}^{k-1-d_c} \binom{N-d}{i_c} P_{\mathrm{cH}_1}^{i_c} P_{\mathrm{eH}_1}^{N-d-i_c}\right] \cdot U(N-d-1) + \sum_{d_c=0}^{k-1+d-N} \binom{d}{d_c} P_{\mathrm{c}_1}^{d_c} P_{\mathrm{e}_1}^{d-d_c}$

and $P_{\mathrm{cH}_1} = 1 - P_{\mathrm{eH}_1}$.

C. Exclusion of Sensors with Noisy Links

For increasing number of noisy links, it is possible to show that the sensor network performance (in terms of probability of decision error at the AP) degrades rapidly. At this point, one might consider an "intelligent" AP, which neglects the decisions of a sensor if the link is noisy. For example, in a wireless sensor network, each sensor could send a pilot symbol to the AP, which, consequently, could determine the status of the corresponding link. Obviously, if some sensors are excluded, there is a loss of information. Therefore, selective elimination of the noisy links will lead to a performance improvement depending on the value of p, i.e., on the probability of bit flipping over noisy links.

In general, given a particular sensor network structure (N sensors and d noisy links), for each value of the sensor SNR it is possible to determine the critical bit-flipping probability which discriminates between (i) using all sensors or (ii) using only the subset of sensors with ideal links. In a network scenario with $N = 5$ sensors and $d = 2$ noisy links, the critical bit flipping probability is shown, as a function of the sensor SNR, in Fig. 4. The diagram has to be interpreted as follows. Given a particular sensor network scenario with a given sensor SNR and a particular value of p (which will depend on the status of the channel between the sensor and the AP), one determines the ($\mathrm{SNR}_{\mathrm{sensor}}$,$p$) network operating point: if this point falls above the critical curve, then the AP should neglect the sensors with bit-flipped links; otherwise, if this point falls below the critical curve, then the AP should use all sensors. For ease of understanding, we have also indicated the critical ($\mathrm{SNR}_{\mathrm{sensor}}$,$p$) operating points corresponding to a probability of error between 10^{-2} and 10^{-6}. For example, consider the sensor SNR corresponding to $P_{\mathrm{e}} = 10^{-3}$: if $p < 0.10$, then using all sensors will lead to a probability of error lower than 10^{-3}; for $p \geq 0.10$, the lowest possible probability of error (equal to 10^{-3}) is obtained by using only the sensors with ideal links.

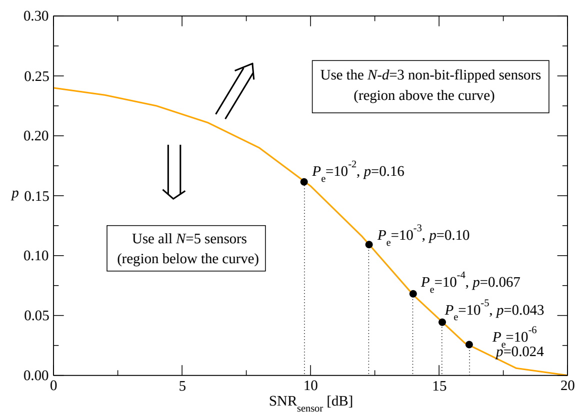

Fig. 4. Critical bit-flipping probability p as a function of the sensor SNR, relative to a sensor network with $N = 5$ sensors and $d = 2$ bit-flipped links. The curve divides two regions: in the above region the best performance is obtained by selecting only the $N - d = 3$ non-bit-flipped links, whereas in the region below the sensor network works better using all $N = 5$ sensors.

We remark that our results in Fig. 4 show that the critical bit-flipping probability decreases for increasing values of the sensor SNR. In other words, whenever sensors are very sensitive (i.e., the sensor SNR is high), then the presence of even a low link noise has a significant impact on the sensor network performance—in fact, the best operating regime is the one corresponding to selective elimination of the sensors

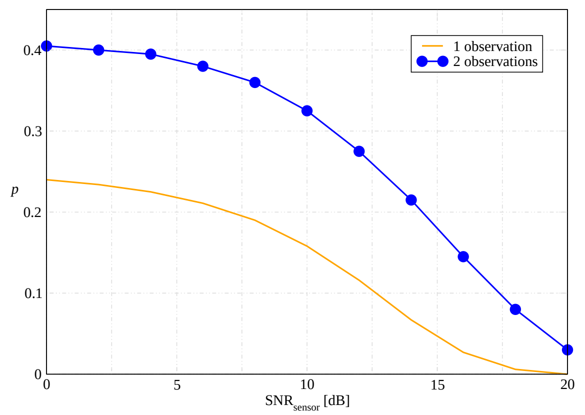

Fig. 5. Critical bit-flipping probability p as a function of the sensor SNR, relative to a sensor network with $N = 5$ sensors and $d = 2$ bit-flipped links with one and two observations per sensor, respectively.

with noisy links. On the constructive side, sensors which are selectively excluded could be temporarily turned off (e.g., by properly estimating the fade duration in a wireless communication scenario), prolonging the sensor network's lifetime.

D. Multiple Observations at the Sensors

In [13], it has been shown that the use of multiple consecutive and independent observations at each sensor has a beneficial effect on the performance, i.e., it reduces the probability of decision error at the AP. While in [13] multiple observations have been considered for sensor networks with *ideal* communication links, we now evaluate the effect of multiple observations in sensor networks with *noisy* communication links. After tedious manipulations (not reported for lack of space), it is possible to extend the previous analysis (conducted in a scenario with single observations at the sensors) and derive analytical expressions for the probability of decision error at the AP. More precisely, in a sensor network scenario with N sensors and d noisy communication links, it is possible to evaluate the critical bit flipping probability which discriminates between (a) using all sensors and (b) discarding the sensors with noisy communication links. The critical bit flipping probability curve is shown in Fig. 5, where, for the sake of comparison, we have also shown the critical bit flipping probability curve of Fig. 4. It is immediate to observe that the critical bit flipping probability increases dramatically when two observations per sensor are used (roughly speaking, it doubles). In other words, our results suggest that use of multiple observations (which comes at the cost of increased delay in the final decision and increased energy consumption at the sensors and AP) makes the sensor network more robust against impairments in the sensor-AP communication links.

IV. Conclusions and Future Work

In this paper, we have considered the problem of distributed detection in sensor networks where some of the communication links between the sensors and the AP may be noisy. First, we have revisited basic principles of distributed detection with binary decisions at the sensors, discussing optimal fusion rules at the AP. Then, we have introduced a simple BSC model for noisy communication links between sensors and AP, and we have analyzed the corresponding network performance, in terms of probability of decision error at the AP. For each value of the sensor SNR, we have shown the existence of a *critical bit-flipping probability*: for values higher than this critical value, network performance is optimized by discarding the decisions coming from sensors with noisy links; for values of p lower than this critical value, network performance is optimized by using the decisions from all sensors. Our results show that the critical bit-flipping probability is a monotonically decreasing function of the sensor SNR.

The different sensor network operating regimes, depending on the number of noisy links and the noise intensity over such links, could be forced by the use of a suitable MAC protocol (with channel sensing) at the AP. We are currently working on the design of a "smart" MAC protocol at the AP. We are also extending our approach to encompass the presence of quantization at the sensors.

References

[1] J. N. Tsitsiklis, *Adv. Statist. Signal Process.*, 1993, vol. 2, ch. Decentralized detection, pp. 297–344, Eds.: H. V. Poor and J. B. Thomas.

[2] I. Y. Hoballah and P. K. Varshney, "An information theoretic approach to the distributed detection problm," *IEEE Trans. Inform. Theory*, vol. 35, no. 5, pp. 988–994, September 1989.

[3] A. Reibman and L. Nolte, "Detection with distributed sensors," *IEEE Trans. Aerosp. Electron. Syst.*, pp. 501–510, December 1981.

[4] ——, "Design and performance comparison of distributed detection networks," *IEEE Trans. Aerosp. Electron. Syst.*, pp. 2474–2478, December 1987.

[5] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors–Part I: Fundamentals," *Proc. IEEE*, vol. 85, no. 1, January 1997.

[6] R. S. Blum, S. A. Kassam, and H. V. Poor, "Distributed detection with multiple sensors–Part II: Advanced topics," *Proc. IEEE*, vol. 85, no. 1, pp. 64–79, January 1997.

[7] C.-Y. Chong and S. P. Kumar, "Sensor networks: evolution, opportunities, and challenges," *Proc. IEEE*, vol. 91, pp. 1247–1256, August 2003.

[8] H. Gharavi and K. Ban, "Multihop sensor network design for wide-band communications," *Proc. IEEE*, vol. 91, no. 8, pp. 1221–1234, August 2003.

[9] S. N. Simic and S. Sastry, "Distributed environmental monitorning using random sensor networks," in *Proc. 2-nd Int. Work. on Inform. Processing in Sensor Networks*, Palo Alto, CA, USA, 2003, pp. 582–592.

[10] T. S. Rappaport, *Wireless Communications. Principles & Practice*, 2nd Edition. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[11] J.-F. Chamberland and V. V. Veeravalli, "Decentralized detection in sensor networks," *IEEE Trans. Signal Processing*, vol. 51, no. 2, pp. 407–416, February 2003.

[12] J. G. Proakis, *Digital Communications*, 4th Edition. New York: McGraw-Hill, 2001.

[13] P. V. S. Althakeem, "Decentralized bayesian detection with feedback," *IEEE Trans. on Systems*, pp. 503–513, July 1996.

[14] H. V. Poor, *An Introduction to Signal Detection and Estimation*. Springer-Verlag, 1994.

[15] R. S. Blum and S. A. Kassam, "Optimum distributed detection of weak signals in dependent sensors," *IEEE Trans. Inform. Theory*, vol. 38, no. 3, pp. 1066–1079, May 1992.

[16] W. Irving and J. Tsitsiklis, "Some properties of optimal thresholds in decentralized detection," *IEEE Trans. Automat. Contr.*, pp. 835–838, 1994.

[17] J. Tsitsiklis, "Decentralized detection by a large number of sensor," *Mathematics of Control, Signals and Systems*, pp. 167–182, 1988.

[18] A. Reibman and L. Nolte, "On determining the design of fusion detection networks," *Proc. of the 27th Conference on Decision and Control*, pp. 2474–2478, December 1988.

[19] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Wiley & Sons, Inc., 1991.