DECENTRALIZED DETECTION IN BINARY DENSE SENSOR NETWORKS: TO TRANSMIT OR NOT TO TRANSMIT

Marcelino Lázaro, Antonio Artés-Rodríguez, and Matilde Sánchez-Fernández

Departmento de Teoría de la Señal y Comunicaciones
Universidad Carlos III de Madrid
Avda. Universidad 30, 28911, Leganés (Madrid) SPAIN
{mlazaro,antonio,mati}@tsc.uc3m.es

ABSTRACT

We consider the problem of binary decentralized detection in large-scale, randomly deployed, dense wireless sensor networks. We compare the performance of a Neyman-Pearson global detector under two different transmission strategies. The first one is based on a censoring scheme in which only the sensors with positive detections try to transmit; the second being the corresponding uncensored one. The aim of the paper is to determine in which situations each strategy needs less energy to achieve a given probability of error.

1. INTRODUCTION

In recent years, much effort has been dedicated to understand the fundamental performance limits in wireless sensor networks. The transport capacity [1, 2, 3] has been studied for different configurations, as well as the asymptotic performance for estimation [2, 3] and detection [4] problems. In this contribution we are interested in comparing the performance limits of two different transmission strategies in binary distributed detection in dense sensor networks.

We consider a large-scale, randomly deployed, dense wireless sensor network which aim is to detect the presence or not of a target. For simplicity, we assume the position of the target to be known. We also assume a many-to-one transmission strategy with no hops. In 1988, Tsitsiklis [5] shown that when the number of sensors is arbitrarily large, the optimal binary decentralized detection is achieved by identical local detection rules, and this result has been recently extended [4] showing that using identical transmitter is also optimal. Based on these results, we assume identical nodes performing local binary decisions. However, we do not restrict the local detectors to be based on a likelihood ratio, thus allowing the use of wide used non-parametric, learning-based local detectors, like in [6]. The behavior of the local detectors will be characterized using a model in which the probability of detection (including false alarms) of the sensor varies as a function p_d of the distance between the sensor and the source or target to be detected, as proposed in [7].

This work is partialy supported by the MCyT of Spain under grant TIC2003-02602.

Under these hypothesis, the aim of this contribution is to compare a recently proposed censoring scheme [8] with the corresponding uncensored one. The censoring scheme is based on the idea that only sensors with positive detections try to transmit their positions. More elaborate censoring schemes has been proposed in the literature [9], but they do not apply in this setting because, as said before, the local decision could not be based on a likelihood ratio. The performance of the network will be assessed by the asymptotic error probability of a Neyman-Pearson detector against power consumption, being the most energy demanding task the wireless transmission, according to [10].

2. PROBLEM STATEMENT AND NOTATION

A set of ℓ sensors are uniformly deployed in a region $\mathcal{D} \in I\!R^2$ of area S. The exploration of $\mathcal{D}$ can produce, potentially, the following data set:

$$\{(\boldsymbol{x}_i, y_i) : i = 1, \ldots, \ell,\ \boldsymbol{x}_i \in \mathcal{D}, y_i \in \{0, 1\}\}.$$

Each pair $(\boldsymbol{x}_i, y_i)$ represents the reading of a sensor located at coordinates $\boldsymbol{x}_i$ that can detect ($y_i = 1$), or not ($y_i = 0$) a specific target. However, for the sake of reducing the energy consumption, and thus enlarging the time life of the network, we assume that a parameter p_s, which defines the probability of sensing, can be dynamically tuned at all sensors. At every sensing instant (automatic or beacon driven), each sensor independently decides to sense with a probability p_s. We denote $\ell_s \leq \ell$ the number of sensors that sense.

After sensing, two different kinds of sensor sets can be defined: the set of ℓ_d sensors with a positive detection ($y = 1$), whose positions are denoted by $\{\boldsymbol{x}_i^d,\ i = 1, \cdots, \ell_d\}$, and the set of ℓ_n sensors with a negative detection ($y = 0$), whose positions are denoted $\{\boldsymbol{x}_i^n,\ i = 1, \cdots, \ell_n\}$. Obviously, $\ell_s = \ell_d + \ell_n$.

We propose two transmission schemes, which will be described in the next sections, where different sets of sensors try to transmit their reading to a fusion center. For both schemes, the medium access and transmission errors are globally modeled through a probability of transmission error, which is denoted by p_e.

The probability of detection of an agent located at coordinates $\boldsymbol{x}^t$, for a sensor at position $\boldsymbol{x}$, is denoted as

$$p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha) = P(Y = 1 | \boldsymbol{X}^t = \boldsymbol{x}^t, \boldsymbol{X} = \boldsymbol{x}),$$

where α denotes the probability of false alarm (PFA) of the detector. This function depends on the nature of the specific detection process, but its general conditions are established in [7].

Given the region $\mathcal{D}$ and the position of a possible agent or target, $\boldsymbol{x}^t$, two hypothesis are defined: H_0, or null hypothesis if a target is not present at $\boldsymbol{x}^t$; H_1 or alternative hypothesis if a target is present. The joint probability density function (pdf) of the observations $(\boldsymbol{X}, Y)$ under each hypothesis is presented in [7].

3. CENSORED TRANSMISSION SCHEME

In this section, we summarize the method proposed in [8] introducing a slight change in notation to accommodate the further inclusion of the uncensored transmission scheme. In this approach, for the sake of saving energy, and taking into account that both sets of positions, $\boldsymbol{x}_i^d$ and $\boldsymbol{x}_j^n$, have discrimination capability, only sensors with a positive detection ($Y = 1$) try to transmit their position $\boldsymbol{x}_i^d$ to the fusion center.

Because of the probability of transmission error, p_e, at the fusion center only the positions of the $\ell_a \leq \ell_d$ sensors that achieved a successful transmission are available. These positions will be labeled $\{\boldsymbol{x}_i^a, i = 1, \cdots, \ell_a\}$. It is also interesting to remark the information that is not available at the fusion center:

- The number of sensors, ℓ_n, that, after sensing, obtain a negative detection ($Y = 0$), and the positions of such sensors, $\{\boldsymbol{x}_i^n, i = 1, \cdots, \ell_n\}$.
- The number of sensors, ℓ_e, that, after sensing and obtaining a positive detection ($Y = 1$), fail to transmit their position to the fusion center, and the positions of such sensors, $\{\boldsymbol{x}_i^e, i = 1, \cdots, \ell_e\}$.

Of course, $\ell_s = \ell_a + \ell_e + \ell_n$ and $\ell_d = \ell_a + \ell_e$. Note that ℓ_a is hypothesis dependent.

3.1. Likelihood ratio

The observations are modeled by means of the random variable $\boldsymbol{\Theta}$

$$\boldsymbol{\Theta} = [\boldsymbol{x}_1, \cdots, \boldsymbol{x}_\ell, \ell_a]^T.$$

This vector includes the possible position of all available sensors and the number of positions available at the fusion center. Without lack of generality, this vector can be ordered as follows

$$\boldsymbol{\Theta} = \left[\boldsymbol{x}_1^a, \cdots, \boldsymbol{x}_{\ell_a}^a, \boldsymbol{x}_{\ell_a+1}, \cdots, \boldsymbol{x}_\ell, \ell_a\right]^T.$$

It must be noted that only the first ℓ_a positions are available at the fusion center. Now, the pdf of the observations under hypothesis H_j is

$$f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_j) = \prod_{i=1}^{\ell_a} f_{X|H,Y}(\boldsymbol{x}_i^a | H_j, 1) \frac{f_{L_a|L,H}(\ell_a|\ell, H_j)}{S^{\ell-\ell_a}},$$

where L_a and L are the random variables modeling the number of sensors that achieved a successful transmission and the total number of deployed sensors, respectively. It is straightforward to obtain $f_{X|H,Y}(\boldsymbol{x}_i|H_j, k)$ by using the definition of the probability of detection $p_d(\boldsymbol{x}^t, \boldsymbol{x}_i, \alpha)$

$$f_{X|H,Y}(\boldsymbol{x}_i|H_1, 1) = \frac{p_d(\boldsymbol{x}^t, \boldsymbol{x}_i, \alpha)}{\int_{\mathcal{D}} p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)\, d\boldsymbol{x}},$$

$$f_{X|H,Y}(\boldsymbol{x}_i|H_1, 0) = \frac{1 - p_d(\boldsymbol{x}^t, \boldsymbol{x}_i, \alpha)}{\int_{\mathcal{D}} [1 - p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)]\ d\boldsymbol{x}},$$

and

$$f_{X|H,Y}(\boldsymbol{x}_i|H_0, 1) = f_{X|H,Y}(\boldsymbol{x}|H_0, 0) = \frac{1}{S}.$$

Therefore

$$f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1) = \prod_{i=1}^{\ell_a} \frac{p_d(\boldsymbol{x}^t, \boldsymbol{x}_i, \alpha)}{\int_{\mathcal{D}} p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)\, d\boldsymbol{x}} \frac{f_{L_a|L,H}(\ell_a|\ell, H_1)}{S^{\ell-\ell_a}},$$

and

$$f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0) = \frac{1}{S^\ell} \cdot f_{L_a|L,H}(\ell_a|\ell, H_0).$$

From these expressions, the likelihood ratio is

$$\begin{aligned}\Gamma &= \frac{f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1)}{f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0)} \\ &= S^{\ell_a} \prod_{i=1}^{\ell_a} \frac{p_d(\boldsymbol{x}^t, \boldsymbol{x}_i, \alpha)}{\int_{\mathcal{D}} p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)\, d\boldsymbol{x}} \frac{f_{L_a|L,H}(\ell_a|\ell, H_1)}{f_{L_a|L,H}(\ell_a|\ell, H_0)}.\end{aligned}$$

The decision is usually given in terms of this ratio

$$\gamma = \ln \Gamma \underset{H_1}{\overset{H_0}{\lessgtr}} \tau.$$

The threshold can be obtained, for instance, by means of asymptotic gaussianity for the Neyman-Pearson criteria (similar to the obtained in [7]).

3.2. Modeling the number of sensors

The distribution of the number of sensors with a successful transmission, L_a, is included in the test. Taking into account that in the exploration region there are ℓ sensors and that each senses with a probability p_s, the distribution of the number of sensors that sense, modeled by the random variable L_s, is a binomial

$$f_{L_s|L}(\ell_s|\ell) = \binom{\ell}{\ell_s} p_s^{\ell_s} (1 - p_s)^{\ell-\ell_s},$$

for $0 \leq \ell_s \leq \ell,\ \ell_s \in \mathbb{Z}$, where L denotes the random variable modeling the number of sensors in $\mathcal{D}$.

Given a probability of transmission error, p_e, the probability of a sensor having a successful transmission is

$$p_t = (1 - p_e) \cdot p_{\mathcal{D}},$$

where $p_{\mathcal{D}}$ is the probability of having a positive detection for a sensor in region $\mathcal{D}$. Obviously, this probability depends on the underlying hypothesis, i.e.

$$p_{t|1} = (1 - p_e) \cdot p_{\mathcal{D}|1},$$

and

$$p_{t|0} = (1 - p_e) \cdot p_{\mathcal{D}|0},$$

where

$$p_{\mathcal{D}|1} = E\left\{p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)\right\} = \frac{1}{S}\int_{\mathcal{D}} p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)\, d\boldsymbol{x},$$

and

$$p_{\mathcal{D}|0} = \alpha.$$

Therefore, taking into account that

$$f_{L_a|L_s}(\ell_a|\ell_s) = \binom{\ell_s}{\ell_a} p_t^{\ell_a}(1 - p_t)^{\ell_s - \ell_a},$$

for $0 \leq \ell_a \leq \ell_s,\ \ell_a \in \mathbb{Z}$, it is straightforward to obtain the distribution of L_a under each hypothesis

$$f_{L_a|L,H}(\ell_a|\ell, H_i) = \sum_{\ell_s=\ell_a}^{\ell} \binom{\ell_s}{\ell_a} p_{t|i}^{\ell_a}\left(1 - p_{t|i}\right)^{\ell_s-\ell_a} \cdot \binom{\ell}{\ell_s} p_s^{\ell_s}\left(1 - p_s\right)^{\ell-\ell_s},$$

for $0 \leq \ell_a \leq \ell,\ \ell_a \in \mathbb{Z}$.

3.3. Bounds on the probability of error

We bound the probability of error in the hypothesis test by using large deviation bounds in the form of error exponents. If ϵ_n is the probability of error (of some kind) obtained with n observations, the error exponent is defined as

$$\lim_{n\to\infty} -\frac{1}{n}\ln \epsilon_n \tag{1}$$

In NP test, the best error exponent is given by the Stein's lemma [11], that applied to our problem says that for any $\alpha_n \in (0, 1)$

$$\lim_{n\to\infty} -\frac{1}{n}\ln \beta_n = D(f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0)\|f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1)), \tag{2}$$

where $D(f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0)\|f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1))$ denotes the Kullback-Leibler (KL) divergence [11] between the probability density functions of the observations under each hypothesis. We will use the notation $D(H_0\|H_1)$ for short.

For the proposed censored transmission scheme, this divergence is (see [8] for details)

$$\begin{aligned} D^c(H_0||H_1) = &\sum_{\ell_a=0}^{\ell} f_{L_a|L,H}(\ell_a|\ell, H_0) \\ &\cdot \left\{ \ln\frac{f_{L_a|L,H}(\ell_a|\ell, H_0)}{f_{L_a|L,H}(\ell_a|\ell, H_1)} - \ell_a \cdot \ln S \right. \\ &\left. - \ell_a \frac{1}{S}\int_{\mathcal{D}} \ln f_{X|H,Y}(\boldsymbol{x}|H_1, 1)\, d\boldsymbol{x} \right\}, \end{aligned}$$

where superindex c denotes "censored".

4. UNCENSORED TRANSMISSION SCHEME

In this section we consider an uncensored transmission scheme, where both sensors with positive ($Y = 1$) and negative ($Y = 0$) detection try to transmit their position ($\boldsymbol{x}_i$) and reading (y_i) to the fusion center. The difference with the approach in [7] is that in this case, a probability of sensing, p_s, is considered (this is a configuration parameter, as in Section 3), and that a probability of transmission error, p_e, is introduced to model the errors in the medium access and in the transmission itself. To summarize, we are facing the following scenario:

- Each of the ℓ sensors, uniformly distributed in the exploration area $\mathcal{D}$, senses with probability p_s, giving $\ell_s \leq \ell$ sensors that sense.
- The number of sensors with a positive detection ($Y = 1$) is denoted as ℓ_d. Because of the probability of transmission error, p_e, only $\ell_{ad} \leq \ell_d$ achieve to successfully transmit their position and reading. These positions, which will be available at the fusion center, are denoted by $\{\boldsymbol{x}_i^{ad}, i = 1, \cdots, \ell_{ad}\}$.
- The number of sensors with a negative detection ($Y = 0$) is denoted as ℓ_n. Because of p_e, only $\ell_{an} \leq \ell_n$ achieve to successfully transmit their position and reading. These positions, which will be also available at the fusion center, are denoted by $\{\boldsymbol{x}_i^{an}, i = 1, \cdots, \ell_{an}\}$.
- The total number of successful transmissions is denoted as ℓ_a, and obviously, $\ell_a = \ell_{ad} + \ell_{an}$.

4.1. Likelihood ratio

Following the same methodology that for the censored scheme, now the observations are modeled by means of the random variable $\boldsymbol{\Theta}$

$$\boldsymbol{\Theta} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_\ell, \ell_{ad}, \ell_{an}].$$

It includes the position of all sensors and the number of available positions for both sensors with positive and negative detections. Without lack of generality, this vector will be ordered as follows

$$\boldsymbol{\Theta} = \left[\boldsymbol{x}_1^{ad}, \cdots, \boldsymbol{x}_{\ell_{ad}}^{ad}, \boldsymbol{x}_1^{an}, \cdots, \boldsymbol{x}_{\ell_{an}}^{an}, \cdots, \boldsymbol{x}_\ell, \ell_{ad}, \ell_{an}\right],$$

and only the values for the first $\ell_a = \ell_{ad} + \ell_{an}$ positions, along with the values of ℓ_{ad} and ℓ_{an}, are available at the fusion center.

The test relies in the likelihood ratio

$$\Gamma = \frac{f_{\boldsymbol{\Theta}|H}(\theta|H_1)}{f_{\boldsymbol{\Theta}|H}(\theta|H_0)}.$$

In this case, the probability density functions of the observations under each hypothesis are, respectively

$$\begin{aligned} f_{\boldsymbol{\Theta}|H}(\theta|H_1) = &\prod_{i=1}^{\ell_{ad}} \frac{p_d(\boldsymbol{x}^t, \boldsymbol{x}_i, \alpha)}{\int_{\mathcal{D}} p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)\, d\boldsymbol{x}} \\ &\cdot \prod_{j=\ell_{ad}+1}^{\ell_{ad}+\ell_{an}} \frac{1 - p_d(\boldsymbol{x}^t, \boldsymbol{x}_j, \alpha)}{\int_{\mathcal{D}} [1 - p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha)]\ d\boldsymbol{x}} \\ &\cdot \frac{f_{L_{ad}, L_{an}|L,H}(\ell_{ad}, \ell_{an}|\ell, H_1)}{S^{\ell-\ell_a}}, \end{aligned}$$

and

$$f_{\boldsymbol{\Theta}|H}(\theta|H_0) = \frac{1}{S^{\ell}} f_{L_{ad}, L_{an}|L,H}(\ell_{ad}, \ell_{an}|\ell, H_0).$$

Here, L_{ad} and L_{nd} denote the random variables modeling, respectively, the number of sensors with a positive detection that transmitted successfully, and the number of sensors with a negative detection that transmitted successfully.

4.2. Modeling the number of sensors

The hypothesis test depends now on the joint probability density (or mass function) $f_{L_{ad}, L_{an}|L,H}(\ell_{ad}, \ell_{an}|\ell, H_i)$, which is given by

$$\begin{aligned} &f_{L_{ad}, L_{an}|L,H}(\ell_{ad}, \ell_{an}|\ell, H_i) = \\ &\sum_{\ell_s=\ell_{ad}+\ell_{an}}^{\ell} \binom{\ell}{\ell_s} p_s^{\ell_s} \left(1 - p_s\right)^{\ell-\ell_s} \\ &\cdot \binom{\ell_s}{\ell_{ad}+\ell_{an}} (1 - p_e)^{\ell_{ad}+\ell_{an}}\, p_e^{\ell_s - \ell_{ad} - \ell_{an}} \\ &\cdot \binom{\ell_{ad}+\ell_{an}}{\ell_{ad}} p_{\mathcal{D}|i}^{\ell_{ad}} \left(1 - p_{\mathcal{D}|i}\right)^{\ell_{an}}. \end{aligned}$$

The first binomial models the number of sensors that sense, given ℓ and the probability of sensing, p_s; the second binomial represents the number of sensors achieving a successful transmission, given the number of sensors that sense (and therefore that try to transmit) and the probability of transmission error, p_e; finally, the third binomial models the number of sensors with a positive detection that achieve a successful transmission, given the total number of successful transmissions and the probability of detection of a sensor in region $\mathcal{D}$ under the underlying hypothesis H_i, $p_{\mathcal{D}|i}$. The value of $p_{\mathcal{D}|i}$ is the same one as in Section 3.

4.3. Bounds on the probability of error

Again, as for the censored scheme, we obtain bounds in the form of error exponents for the Neyman-Pearson test. The Kullback-Leibler divergence between the pdf's under each hypothesis is, in this case

$$\begin{aligned} &D^u(H_0||H_1) = \\ &\sum_{\ell_{ad}=0}^{\ell} \sum_{\ell_{an}=0}^{\ell-\ell_{ad}} f_{L_{ad}, L_{an}|L,H}(\ell_{ad}, \ell_{an}|\ell, H_0) \\ &\cdot \Bigg\{ \ln \frac{f_{L_{ad}, L_{an}|L,H}(\ell_{ad}, \ell_{an}|\ell, H_0)}{f_{L_{ad}, L_{an}|L,H}(\ell_{ad}, \ell_{an}|\ell, H_1)} \\ &- (\ell_{ad} + \ell_{an}) \cdot \ln S \\ &- \frac{\ell_{ad}}{S} \int_{\mathcal{D}} \ln f_{X|H,Y}(\boldsymbol{x}|H_1, 1)\ d\boldsymbol{x} \\ &- \frac{\ell_{an}}{S} \int_{\mathcal{D}} \ln f_{X|H,Y}(\boldsymbol{x}|H_1, 0)\ d\boldsymbol{x} \Bigg\}, \end{aligned}$$

where superindex u denotes "uncensored".

5. CENSORED VS. UNCENSORED: RESULTS USING THE "SPANISH HAT" PROBABILITY OF DETECTION

We want to determine which one is the best option in terms of the probability of error per unit of spent energy in the asymptotic case, where the probability of error for a NP test is related through (1) and (2) with $D(H_0||H_1)$.

To analyze and compare the performance of the proposed tests, we have used the so-called "spanish hat" probability of detection. This model, which can be seen as a first order approximation for any probability of detection, is defined as

$$p_d(\boldsymbol{x}^t, \boldsymbol{x}, \alpha) = \begin{cases} (1-\beta) & \text{si } ||\boldsymbol{x}^t - \boldsymbol{x}||_2 < r_o \\ \alpha & \text{other case} \end{cases},$$

where r_o is the sensor range, and β is the probability of misdetection. For the sake of simplicity, we have defined circular exploration areas $\mathcal{D}$, with radius R, centered at the position of the possible agent, $\boldsymbol{x}^t$.

It is straightforward to obtain the expressions involved in the KL divergence for both tests that depend on the probability of detection function for the "spanish hat" model. For instance,

$$p_{\mathcal{D}|1} = \begin{cases} (1-\beta) & \text{if } R \leq r_o \\ \dfrac{r_o^2(1-\beta) + (R^2 - r_o^2)\alpha}{R^2} & \text{if } R > r_o \end{cases},$$

and,

$$p_{\mathcal{D}|0} = \alpha.$$

The expressions for $\int_{\mathcal{D}} \ln f_{X|H,Y}(\boldsymbol{x}|H_1, 1)\ d\boldsymbol{x}$ and for $\int_{\mathcal{D}} \ln f_{X|H,Y}(\boldsymbol{x}|H_1, 0)\ d\boldsymbol{x}$ are not included because of the space constraints, but they are straightforward.

First, we compare the performance/energy ratio achieved for each transmission strategy. For the sake of simplicity, we consider that energy is proportional to the number of sensors trying to transmit their data to the fusion center. We will label this number L^t. For the uncensored scheme, its expected value is given by

$$E^u\left\{L^t\right\} = \ell \cdot p_s.$$

For the censored scheme, the expected value of L^t depends on the underlying hypothesis as

$$E^c_{H_0}\left\{L^t\right\} = \ell \cdot p_s \cdot \alpha, \quad E^c_{H_1}\left\{L^t\right\} = \ell \cdot p_s \cdot p_{\mathcal{D}|1}.$$

Figure 1 plots the $D(H_0\|H_1)$ divergence, normalized by the expected value of L^t, for the uncensored scheme and for the censored one under both hypothesis. The following parameters have been considered: $\alpha = 0.1$, $\beta = 0.1$, $r_o = 1$, and 100π sensors per unity area.

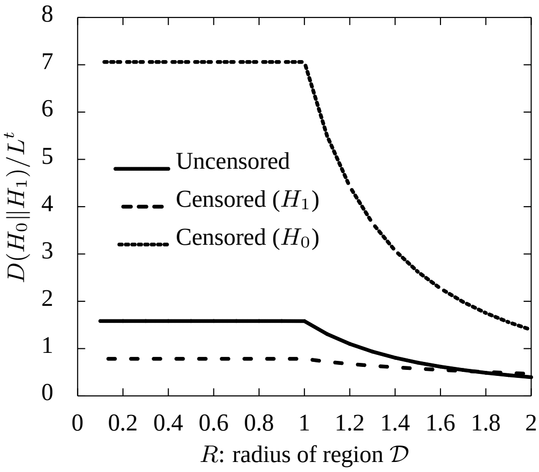

Fig. 1. Divergence $D(f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_0)\|f_{\boldsymbol{\Theta}|H}(\boldsymbol{\theta}|H_1))$ normalized per energy spent (proportional to the number of sensors trying to transmit) for the "spanish hat" model and a circular region $\mathcal{D}$ centered at the position of the possible agent to be detected, $\boldsymbol{x}^t$, as a function of the radius, R, of region $\mathcal{D}$.

The first noticeable result is that this plot allows to define the optimal size for region $\mathcal{D}$. As expected, the optimal size is limited in this case by the sensor range. If the radius R is larger than r_o, the $D(H_0\|H_1)/L^t$ ratio decays. Constrained to regions with $R \leq r_o$, the uncensored scheme provides a better ratio than the censored scheme under hypothesis H_1. However, under hypothesis H_0 a small number of sensors will obtain a positive reading ($Y = 1$), and therefore the $D(H_0\|H_1)/L^t$ ratio for the censored scheme in this case is better than for the uncensored scheme.

From this results, it is clear than the optimal strategy will depend on the priors for each hypothesis, π_1 and π_0. The censored strategy will be better than the uncensored one for low probabilities of the hypothesis H_1, i.e., for low values of π_1, while the uncensored will be better for high values of π_1. There exists a threshold value which defines the value of π_1 where both schemes achieve the same performance/energy ratio. For the spanish hat probability of detection with regions such that $R < r_o$, this threshold is given by

$$\pi^{th} = \frac{\frac{D^c(H_0||H_1)}{D^u(H_0||H_1)} - \alpha}{1 - \alpha - \beta}. \tag{3}$$

If $\pi_1 > \pi^{th}$, then the uncensored scheme provides the best performance/energy ratio. For $\pi_1 < \pi^{th}$, the censored scheme is more appropriate. It must be remarked that the ratio $\frac{D^c(H_0||H_1)}{D^u(H_0||H_1)}$ in (3) implicitly depends on α and β, as well as on p_s, p_e or ℓ.

Figures 2 and 3 plot this threshold as a function of α (or β) for different values of β (or α), respectively. It

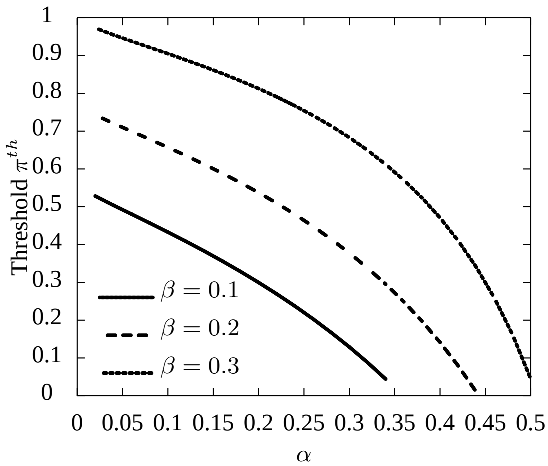

Fig. 2. Threshold π^{th} as a function of the probability of false alarm, α.

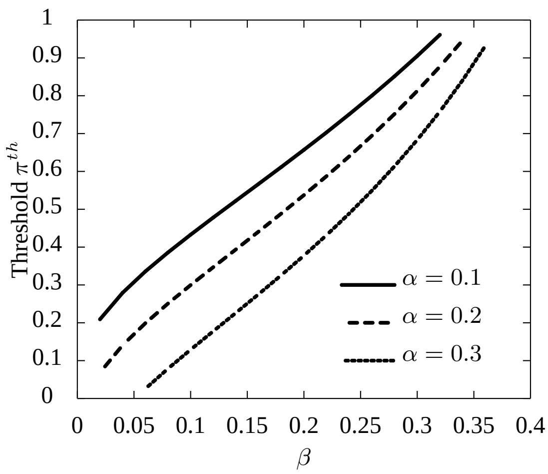

Fig. 3. Threshold π^{th} as a function of the probability of misdetection, β.

can be seen that the threshold is an increasing function of β and a decreasing function of α. Moreover, in some cases if α is increased enough, the expression (3) can take a negative value. When this happens, the uncensored scheme is always the best option. A similar behavior can be observed at other cases if β is increased. In this case the threshold can take a value greater than one. In these cases, the censored scheme is always the best choice. In any case, taking values for α and β in the usual range of practical sensors, the threshold value does not exceed the above mentioned limit values.

Finally, the obtained results can be used to perform the design of several parameters of the network, like p_s, or the parameters related with p_e (like transmission power, codification rule, medium access strategy, etc.). Figure 4 plots the divergence for the censored scheme as a function of p_s and p_e. Once a given discrimination level is necessary to achieve the desired behavior, this plot can be used to select the above mentioned parameters.

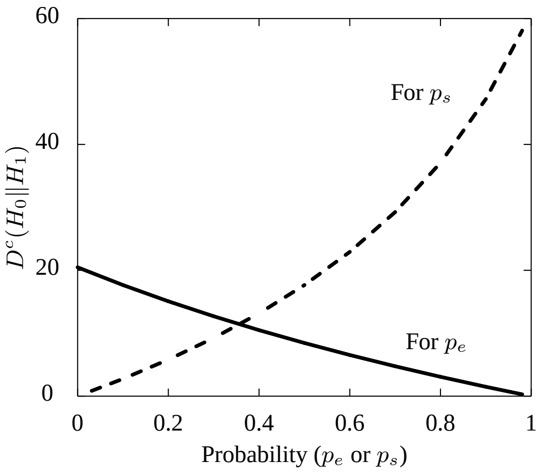

Fig. 4. Divergence $D^c(H_0\|H_1)$ for the censored scheme using the spanish hat model and a circular region $\mathcal{D}$ centered at the position, $\boldsymbol{x}^t$, of the possible agent to be detected, as a function of the sensing probability, p_s, and the probability of transmission error, p_e.

6. CONCLUSIONS

In this contribution, two transmission schemes have been analyzed. Asymptotic measurements for the probability of error, in the shape or error exponents, for the detection hypothesis test, have been obtained and compared under censored and uncensored transmission schemes in binary sensor networks. The provided results allow to decide the best option, in the sense of the one minimizing the probability or error/energy ratio. The best option depends on the priors of both hypothesis, and on the characteristics of the local detector. The expression of the threshold defining the best option has been provided for a simplified probability of detection, named the spanish hat model.

The provided results are helpful to design several network parameters, as the probability of sensing, the number of sensors per area, or the parameters related with the probability of transmission error.

However, further work must be done to complete these results in several ways, for instance:

- To introduce a more realistic model of the transmission errors, with a dependence on the transmission load and the MAC;
- to consider the position of the target to be unknown;
- the extension to more than one target;
- the extension to include also Bayes detectors.

7. REFERENCES

[1] F. Xue, L. L. Xie, and P.R. Kumar, "The transport capacity of wireless networks over fading channels," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 834–847, Mar. 2005.

[2] D. Marco, E. J. Duarte-Melo, M. Liu, and D. L. Neuhoff, "On the many-to-one transport capacity of a dense wireless sensor network and the compressibility of its data," in *International Workshop on Information Processing in Sensor Networks*, Berkeley, CA, Apr. 2003.

[3] H. El Gamal, "On the scaling laws of dense wireless sensor networks: the data gathering channel," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 1229–1234, Mar. 2005.

[4] J. F. Chamberland and V. V. Veeravalli, "Asymptotic results for decentralized detection in power constrained wireless sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1007–1015, Aug. 2004.

[5] John N. Tsitsiklis, "Decentralized detection by a large number of sensors," *Mathematics of Control, Signals, and Systems*, vol. 1, pp. 167–182, 1988.

[6] X. Nguyen, M. J. Wainwright, and M. I. Jordan, "Nonparametric decentralized detection using kernel methods," *IEEE Transactions on Signal Processing*, In press.

[7] Antonio Artés-Rodríguez, "Decentralized detection in sensor networks using range information," in *Proceedings of the ICASSP 2004*, Montreal, Canada, May 2004.

[8] A. Artés-Rodríguez, M. Lázaro, and M. Sánchez-Fernández, "Decentralized detection in sensor networks with censored transmissions," in *Proceedings of the ICASSP 2005*, Philadelphia, USA, Mar. 2005.

[9] C. Rago, P. Willett, and Y. Bar-Shalom, "Censoring sensors: a low communication-rate scheme for distributed detection," *IEEE Trans. on AES*, vol. 32, no. 2, pp. 554–568, Apr. 1996.

[10] L. Doherty, B. A. Warneke, B. Boser, and K. S. J. Pister, "Energy and performance considerations for Smart Dust," *Int. Journal of Parallel and Distributed Sensor Networks*, vol. 4, no. 3, Dec. 2001.

[11] T.M. Cover and J.A. Thomas, *Elements of information theory*, J. Wiley & Sons, New York, NY, 1991.